Assignment #10

Due on Friday, February 27, 2009

Read Section 3.5 on *Dimension* in Messer (pp. 114–121).

Background and Definitions

(Definition of dimension of a subspace of $\mathbb{R}^n$*).* Let W be a subspace of $\mathbb{R}^n$. The dimension of W, denoted by $\dim(W)$, is the number of vectors in any basis for W.

Do the following problems

1. Let
$$W_1 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; x + y - z = 0 \right\} \text{ and } W_2 = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; x + 2y + z = 0 \right\}.$$
   Find a bases for W_1 and W_2 and compute $\dim(W_1)$ and $\dim(W_2)$.

2. Let W_1 and W_2 be as defined in Problem 1. Find a basis for $W_1 \cap W_2$ and compute $\dim(W_1 \cap W_2)$.

3. Let W_1 and W_2 be as defined in Problem 1. Find a basis for $W_1 + W_2$ and compute $\dim(W_1 + W_2)$.

   Use the results of Problems 1 and 2 to verify that
$$\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2).$$

4. Let $A = \begin{pmatrix} 1 & -2 & -3 & 0 \\ -1 & 0 & 2 & 1 \\ 1 & 4 & 0 & -3 \end{pmatrix}$.
   (a) Find a basis for the column space, C_A, of the matrix A and compute $\dim(C_A)$.
   (b) Find a basis for the null space, N_A, of the matrix A and compute $\dim(N_A)$.
   (c) Compute $\dim(N_A) + \dim(C_A)$. What do you observe?

5. Let A denote the matrix defined in the previous problem. Consider the rows of A as row vectors in $\mathbb{R}^4$, and let R_A denote the span of the rows of the matrix A. Find a basis for R_A, and compute $\dim(R_A)$. What do you find interesting about $\dim(R_A)$ and $\dim(C_A)$, which was computed in the previous problem.