Assignment #16

Due on Wednesday, April 1, 2009

Read Section 5.2 on *Inverses* in Messer (pp. 184–190).

Do the following problems

1. Let A denote an $m \times n$ matrix and let $\{e_1, e_2, \ldots, e_n\}$ denote the standard basis in $\mathbb{R}^n$.

   (a) Prove that if A has a left–inverse, B, then the set $\{Ae_1, Ae_2, \ldots, Ae_n\}$ is a linearly independent subset of $\mathbb{R}^m$.

   (b) Prove that if A has a right–inverse, C, then the set $\{Ae_1, Ae_2, \ldots, Ae_n\}$ spans $\mathbb{R}^m$.

2. Assume $A \in \mathbb{M}(n, n)$ is invertible. Prove that the columns of A form a basis for $\mathbb{R}^n$.

3. Let A and B denote $n \times n$ matrices. Prove that if A and B are invertible, then so is their product, AB, and compute $(AB)^{-1}$ in terms of A^{-1} and B^{-1}.

4. An $n \times n$ matrix, E, is said to be an **elementary matrix** if it is the result of performing an elementary row operation on the $n \times n$ identity matrix, I. Consider the following 3×3 matrices

$$E_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 1 & 0 & 0 \\ c & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \text{and} \quad E_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & d \end{pmatrix},$$

   where c and d are scalars with $d \neq 0$.

   (a) Explain why E_1, E_2 and E_3 are elementary matrices.

   (b) Show that E_1, E_2 and E_3 are invertible and compute there inverses. Are the inverses also elementary matrices?

   (c) Given an 3×3 matrix A, what is the result of multiplying A by E_1, E_2 and E_3 on the left; that is, what are E_iA, for $i = 1, 2, 3$?

5. Let $A \in \mathbb{M}(n, n)$ be invertible. Prove that the transpose, A^T, of A is also invertible and compute its inverse. Deduce, therefore, that, if A is invertible, then the rows of of A are linearly independent.