Assignment #17

Due on Friday, April 3, 2009

Read Section 5.2 on *Inverses* in Messer (pp. 184–190).

Background and Definitions

(Elementary Matrix). A matrix, $E \in \mathbb{M}(m,m)$, which is obtained from the $n \times n$ identity matrix, I, by performing a single elementary row operation on I is called an **elementary matrix**.

(Row Equivalence). A matrix $A \in \mathbb{M}(m,n)$ is said to be **row equivalent** to a matrix $B \in \mathbb{M}(m,n)$ if there exist elementary matrices, $E_1, E_2, \ldots, E_k \in \mathbb{M}(m,m)$ such that

$$E_k E_{k-1} \cdots E_2 E_1 A = B.$$

(Singular Matrix). A matrix $A \in \mathbb{M}(m,n)$ is said to be **singular** if the equation $Ax = \mathbf{0}$ has non–trivial solutions.

(Nonsingular Matrix). A matrix $A \in \mathbb{M}(m,n)$ is said to be **nonsingular** if the equation $Ax = \mathbf{0}$ has only the trivial solution.

Do the following problems

1. Prove that if $ad - bc \neq 0$, then the matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible and compute A^{-1}.

2. Let A, B and C denote matrices in $\mathbb{M}(m,n)$. Prove the following statements regarding row equivalence.

   (a) A is row equivalent to itself.

   (b) If A is row equivalent to B, then B is row equivalent to A.

   (c) If A is row equivalent to B and B is row equivalent to C, then A is row equivalent to C.

   Note: these properties are usually known as *reflexivity, symmetry* and *transitivity,* respectively, and they define an *equivalence relation.*

3. Use Gaussian elimination to determine whether the matrix

$$A = \begin{pmatrix} 1 & -4 & 1 \\ 0 & 3 & -1 \\ -3 & 0 & 1 \end{pmatrix}$$

is invertible or not. If A is invertible, compute its inverse.

4. Let A denote an $m \times n$ matrix.

   (a) Show that if $m < n$, then A is singular.

   (b) Prove that A is singular if and only if the columns of A are linearly dependent in $\mathbb{R}^m$.

5. Let A denote an $n \times n$ matrix. Prove that A is invertible if and only if A is nonsingular.