Assignment #3

Due on Wednesday, February 4, 2009

Read Section 3.3 on *Linear Independence* in Messer (pp. 103–109).

Do the following problems

1. Consider the vectors v_1, v_2 and v_3 in $\mathbb{R}^3$ given by

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} \quad \text{and} \quad v_3 = \begin{pmatrix} 0 \\ -4 \\ 3 \end{pmatrix}.$$

   (a) If possible, write the vector v_3 as a linear combination of v_1 and v_2.

   (b) Determine whether the set $\{v_1, v_2, v_3\}$ spans $\mathbb{R}^3$.

2. Let v_1, v_2 and v_3 be as given in the previous problem. Find a linearly independent subset of $\{v_1, v_2, v_3\}$ which spans span$\{v_1, v_2, v_3\}$.

3. Show that the set $\left\{\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}\right\}$ is a linearly independent subset of $\mathbb{R}^3$.

4. Determine whether the set $\left\{\begin{pmatrix} 2 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -1 \\ -2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \\ 0 \end{pmatrix}\right\}$ is a linearly independent subset of $\mathbb{R}^4$.

5. Show that $\left\{\begin{pmatrix} 2 \\ 2 \\ 6 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 3 \\ -2 \end{pmatrix}\right\}$ is a linearly dependent subset of $\mathbb{R}^4$. Write one of the vectors in the set as a linear combination of the other three. Show that the remaining three vectors form a linearly independent subset of $\mathbb{R}^4$.