Assignment #4

Due on Friday, February 6, 2009

Read Section 1.8 on *Subspaces* in Messer (pp. 39–44).

Background and Definitions

(Definition of Subspace of $\mathbb{R}^n$*).* A non–empty subset, W, of Euclidean space, $\mathbb{R}^n$, is said to be a **subspace** of $\mathbb{R}^n$ iff

(i) $v, w \in W$ implies that $v + w \in W$ (closure under vector addition); and

(ii) $t \in \mathbb{R}$ and $v \in W$ implies that $tv \in W$ (closure under scalar multiplication).

Do the following problems

1. Let $S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; x \geqslant 0, y \geqslant 0 \right\}$. Show that S is closed under vector addition in $\mathbb{R}^2$. Explain why S is not a subspace of $\mathbb{R}^2$.

2. Let $a_1, a_2, b_1, b_2, c_1, c_2$ be real constants. Let W be the solution set of the homogeneous system
$$\left\{ \begin{array}{rcl} a_1x_1 + b_1x_2 + c_1x_3 & = & 0 \\ a_2x_1 + b_2x_2 + c_2x_3 & = & 0. \end{array} \right.$$
Prove that W is a subspace of $\mathbb{R}^3$.

3. Let $L = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{R}^2 \;\middle|\; y = 2x + 1 \right\}$. Determine whether or not L is a subspace of $\mathbb{R}^2$.

4. Let W be a subspace of $\mathbb{R}^n$. Use the definition of subspace to prove the following statements.

   (a) If $v \in W$, then W must also contain the additive inverse of v.

   (b) W contains the zero vector.

5. Given two subsets A and B of $\mathbb{R}^n$, the **intersection** of A and B, denoted by $A \cap B$, is the set which contains all vectors that are both in A and B; in symbols,
$$A \cap B = \{v \in \mathbb{R}^n \mid v \in A \text{ and } v \in B\}.$$

   (a) Prove that $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

   (b) Prove that if W_1 and W_2 are two subspaces of $\mathbb{R}^n$, then the intersection $W_1 \cap W_2$ is a subspace of $\mathbb{R}^n$ which is contained in both W_1 and W_2.