Assignment #6

Due on Monday, February 16, 2009

Read Section 1.8 on *Subspaces* in Messer (pp. 39–44).

Read Section 3.2 on *Span* in Messer (pp. 97–102).

Read Section 3.3 on *Linear Independence* in Messer (pp. 103–109).

Background and Definitions

(Solution Space of Homogeneous Linear Systems). The set of solutions of the homogeneous system of linear equations

$$\left\{\begin{array}{rcl} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & = & 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & = & 0 \\ \vdots & = & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & = & 0, \end{array}\right. \tag{1}$$

where a_{1j}, for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$, are scalars, is a subspace of $\mathbb{R}^n$. We call it the **solution space** of the system (1).

Do the following problems

1. Let W denote the solution space of the equation

$$3x_1 + 8x_2 + 2x_3 - x_4 + x_5 = 0$$

   Find a linearly independent subset, S, of $\mathbb{R}^5$ such that $W = \text{span}(S)$.

2. Let W denote the solution space of the system

$$\left\{\begin{array}{rcl} x_1 - 2x_2 - x_3 & = & 0 \\ 2x_1 - 3x_2 + x_3 & = & 0. \end{array}\right.$$

   Find a linearly independent subset, S, of $\mathbb{R}^3$ such that $W = \text{span}(S)$.

3. In the following system, find the value or values of λ for which the system has nontrivial solutions. In each case, give a a linearly independent subset of $\mathbb{R}^2$ which generates the solution space.

$$\left\{\begin{array}{rcl} (\lambda - 3)x + \quad y & = & 0 \\ x + (\lambda - 3)y & = & 0 \end{array}\right.$$

4. Let $v \in \mathbb{R}^n$ and S be a subset of $\mathbb{R}^n$.

   (a) Show that the set $\{v\}$ is linearly independent if and only if $v \neq \mathbf{0}$.

   (b) Show that if $\mathbf{0} \in S$, then S is linearly dependent.

5. Let v_1 and v_2 be vectors in $\mathbb{R}^n$, and let c be a scalar.

   (a) Show that $\{v_1, v_2\}$ is linearly independent if and only if $\{v_1, cv_1 + v_2\}$ is also linearly independent.

   (b) Show that
   $$\text{span}(\{v_1, v_2\}) = \text{span}(\{v_1, cv_1 + v_2\}).$$