Assignment #7

Due on Wednesday, February 18, 2009

Read Section 2.2 on *Gaussian Elimination* in Messer (pp. 69–74).

Read Section 2.3 on *Solving Linear Systems* in Messer (pp. 76–79).

Read Section 3.2 on *Span* in Messer (pp. 97–102).

Read Section 3.3 on *Linear Independence* in Messer (pp. 103–109).

Background and Definitions

(Fundamental Theorem of Homogeneous Linear Systems; Theorem 2.6 in Messer, pg. 78). A homogeneous system of m linear equations in n unknowns,

$$\left\{\begin{array}{rcl} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n & = & 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n & = & 0 \\ \vdots & = & \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n & = & 0, \end{array}\right. \tag{1}$$

with $n > m$ has at least one nontrivial solution.

Do the following problems

1. Prove that if the homogeneous system in (1) has a nontrivial solution, then it has infinitely many solutions.

2. Consider the vectors v_1, v_2, v_3 and v_4 in $\mathbb{R}^4$ given by

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 2 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 2 \\ -1 \\ 1 \\ -1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 0 \\ -1 \\ 3 \\ -5 \end{pmatrix}, \quad \text{and} \quad v_4 = \begin{pmatrix} 1 \\ -3 \\ 0 \\ 1 \end{pmatrix}.$$

   Determine whether the set $\{v_1, v_2, v_3, v_4\}$ is linearly independent; if not, find a linearly independent subset of $\{v_1, v_2, v_3, v_4\}$ which spans $\text{span}\{v_1, v_2, v_3, v_4\}$.

3. Let

$$W = \text{span}\left(\left\{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 4 \\ 3 \end{pmatrix}\right\}\right).$$

   Find a linearly independent subset of W which spans W.

4. Let W denote the solution space of the system

$$\left\{\begin{array}{lcl} 3x_1 - 2x_2 - 2x_3 - x_4 + x_5 & = & 0 \\ x_1 - 3x_2 - 2x_5 & = & 0 \\ 2x_2 + x_3 + 2x_4 - x_5 & = & 0 \\ -x_1 + x_2 - x_3 + x_4 - x_5 & = & 0. \end{array}\right.$$

   Find a linearly independent subset, S, of $\mathbb{R}^5$ such that $W = \text{span}(S)$.

5. Determine whether or not the vector $\begin{pmatrix} 4 \\ 7 \\ 7 \\ 4 \end{pmatrix}$ lies in the span of the set

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 3 \\ -2 \end{pmatrix} \right\}.$$