Assignment #11

Due on Wednesday, March 24, 2010

Read Chapter 5 on *Upper Bounds and Suprema,* pp. 80–85, in Schramm's text.

Read Section 9.2 on *Convergence,* pp. 147–150, in Schramm's text.

Do the following problems

1. Use the fact that $\sqrt{2} = \sup\{q \in \mathbb{Q} \mid q > 0 \text{ and } q^2 < 2\}$ to prove that there exists a sequence of rational numbers, (q_n), such that
$$\lim_{n\to\infty} q_n = \sqrt{2}.$$

2. Let (ε_n) denote a sequence of positive numbers which converges to 0. Let (x_n) be a sequence of real numbers and $x \in \mathbb{R}$. Assume there exists $N_1 \in \mathbb{N}$ such that
$$|x_n - x| \leqslant \varepsilon_n \quad \text{for all} \quad n \geqslant N_1.$$
Prove that (x_n) converges to x.

3. Let $x_n = \dfrac{1}{n!}$ for $n \in \mathbb{N}$. Prove that the sequence (x_n) converges to 0.

4. Let (x_n) be a sequence of real numbers converging to $a \neq 0$. Prove that there exists $N \in \mathbb{N}$ such that
$$n \geqslant N \Rightarrow |x_n| > \frac{|a|}{2}.$$

5. Let (x_n) be a sequence of non–zero, real numbers converging to $a \neq 0$. Prove that the set $A = \left\{ \dfrac{1}{x_n} \;\middle|\; n \in \mathbb{N} \right\}$ is bounded.