Assignment #12

Due on Wednesday, March 31, 2010

Read Section 9.2 on *Convergence,* pp. 147–150, in Schramm's text.

Read Section 9.3 on *Convergent Sequences,* p. 150, in Schramm's text.

Read Section 9.4 on *Sequences and Order,* pp. 151–152, in Schramm's text.

Read Section 9.5 on *Sequences and Algebra,* p. 153, in Schramm's text.

Do the following problems

1. Let (x_n) denote a sequence of real numbers. Prove that if $\lim_{n\to\infty} |x_n| = 0$, then (x_n) converges to 0.

2. Let $x_n = \dfrac{(-1)^{n+1}}{\sqrt{n}}$ for all $n \in \mathbb{N}$. Prove that (x_n) converges to 0.

3. Let (x_n) denote a sequence of real numbers.

   (a) Prove that if (x_n) converges then $(|x_n|)$ converges.

   (b) Show that the converse of the statement in part (a) is not true.

4. Let (x_n) and (y_n) denote two convergent sequences. Suppose there exists some $N_1 \in \mathbb{N}$ such that
$$n \geqslant N_1 \Rightarrow x_n \leqslant y_n.$$
   Prove that
$$\lim_{n\to\infty} x_n \leqslant \lim_{n\to\infty} y_n.$$

5. Let (x_n) and (y_n) denote sequences of real numbers. Determine whether the following statements are true or false. If false, provide a counterexample. If true provide an argument to establish the statement as true.

   (a) If (x_n) converges and $(x_n \cdot y_n)$ converges, then (y_n) converges.

   (b) If (x_n) converges and $(x_n + y_n)$ converges, then (y_n) converges.