Assignment #13

Due on Monday, April 12, 2010

Do the following problems.

1. Let (x_n) denote a sequence of real numbers and (x_{n_k}) denote a subsequence of (x_n).

   (a) Prove that if (x_n) converges then (x_{n_k}) converges.

   (b) Show that the converse of the statement proved in part (a) is not true.

In Problems 2-5, you will prove the Binomial Theorem: for any $a, b \in \mathbb{R}$,

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}, \tag{1}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad \text{for } k = 0, 1, 2, \ldots, n,$$

are called the binomial coefficients.

2. Use the formula $(k+1)! = (k+1) \cdot k!$ to establish that $0! = 1$, and compute $\binom{m}{0}$ and $\binom{m}{m}$ for all $m \in \mathbb{N}$.

3. Prove that $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$ for all $n \in \mathbb{N}$ and all $k = 1, \ldots, n$.

4. Use induction to prove that, for any real number, x,

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k. \tag{2}$$

5. Use the expansion in (2) to deduce the expansion in (1) for any real numbers a and b.