Assignment #15

Due on Wednesday, April 21, 2010

Do the following problems.

1. Let m denote a natural number and define $x_n = \dfrac{1}{n^m}$ for all $n \in \mathbb{N}$. Prove that (x_n) converges to 0 as $n \to \infty$.

2. Let q denote a positive rational number and define $x_n = \dfrac{1}{n^q}$ for all $n \in \mathbb{N}$. Prove that (x_n) converges to 0 as $n \to \infty$.

3. Let (x_n) denote a sequence of nonnegative real numbers. Suppose that (x_n) converges to a as $n \to \infty$. Prove that $a \geqslant 0$ and that $(\sqrt{x_n})$ converges to $\sqrt{a}$.

4. Let $x_n = \sqrt{\dfrac{n+1}{n}}$ for all $n \in \mathbb{N}$. Prove that the sequence (x_n) converges and compute its limit.

5. Let $x_n = \sqrt{n^2+n} - n$ for all $n \in \mathbb{N}$. Determine if the sequence (x_n) converges or not. It it converges, compute its limit.

   Suggestion: Consider the product $(\sqrt{n^2+n} + n)x_n$ for each $n \in \mathbb{N}$.