Assignment #5

Due on Wednesday, February 10, 2010

Read Handout #2 on *The Real Numbers System Axioms.*

Read Section 4.6 on *Ordered Fields* on pp. 63–66 in Schramm's text.

Read Section 4.7 on *Absolute Value and Distance* on pp. 68–68 in Schramm's text.

Do the following problems

Use the order and field axioms in Handout #2 to prove the following:

1. Let a, b, c and d denote real numbers.
   Prove that if $a < b$ and $c < d$, then $a + c < b + d$.

2. For any real number a, show that $|-a| = |a|$.

3. Let a and b denote real numbers with $b \neq 0$. Show that
$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}.$$

4. Prove that $|a + b + c| \leq |a| + |b| + |c|$ for all real numbers a, b and c.

5. Use induction on n to prove that
$$2^n > n \quad \text{for all} \quad n \in \mathbb{N}.$$