Assignment #6

Due on Monday, February 15, 2010

Read Handout #2 on *The Real Numbers System Axioms.*

Read Section 4.6 on *Ordered Fields* on pp. 63–66 in Schramm's text.

Read Section 4.7 on *Absolute Value and Distance* on pp. 68–68 in Schramm's text.

Do the following problems

1. Let $x \in \mathbb{R}$. Prove that $0 < x \leqslant 1$ implies that $x^2 \leqslant x$.

2. Let a and b denote real numbers. Use the triangle inequality to prove that
$$||a| - |b|| \leqslant |a - b|.$$

3. Let a and b denote **positive** real numbers. Start with the true statement
$$(a - b)^2 \geqslant 0$$
to prove the inequality
$$ab \leqslant \frac{a^2 + b^2}{2}.$$
Prove that equality holds if and only if $a = b$.

4. Given a real number x, denote by $\max\{x, 0\}$ the larger of x and 0. Prove that
$$\max\{x, 0\} = \frac{x + |x|}{2}.$$

5. Let x and $\max\{x, 0\}$ be as in the previous problem. Denote by $\min\{x, 0\}$ the smaller of x and 0. Prove that
$$\min\{x, 0\} = -\max\{-x, 0\},$$
and use this result to derive a formula for $\min\{x, 0\}$ analogous to that for $\max\{x, 0\}$ proved in the previous problem.