Assignment #8

Due on Wednesday, March 3, 2010

Read Handout #2 on *The Real Numbers System Axioms.*

Read Section 4.6 on *Ordered Fields* on pp. 63–66 in Schramm's text.

Read Chapter 5 on *Upper Bounds and Suprema,* pp. 80–85, in Schramm's text.

Do the following problems

1. Let $a, b \in \mathbb{R}$. Show that if $a < b + \dfrac{1}{n}$ for all $n \in \mathbb{N}$, then $a \leqslant b$.

2. Show that $\sup\{t \in \mathbb{R} \mid t < a\} = a$ for each $a \in \mathbb{R}$.

3. A subset, A, of the real numbers is said to be **bounded** if there exists a positive real number, M, such that
$$|a| \leqslant M \quad \text{for all} \quad a \in A.$$
Prove that A is bounded if and only if A is bounded above and below.

4. For real numbers a and b with $a < b$, $[a, b]$ denotes the closed, bounded, interval from a to b; that is,
$$[a, b] = \{x \in \mathbb{R} \mid a \leqslant x \leqslant b\}.$$
Assume that $A \subseteq \mathbb{R}$ is nonempty and bounded. Prove that
$$A \subseteq [\inf(A), \sup(A)].$$

5. Let A denote a nonempty and bounded subset of the real numbers. Prove that if I is a closed interval with $A \subseteq I$, then
$$[\inf(A), \sup(A)] \subseteq I.$$