# Exam 1 (Part II)

Friday, February 19, 2010

Name: ________________________________

This is the out–of–class portion of Exam 1. There is no time limit for working on the following two problems. You are only allowed to consult Handout #2 on the axioms of the real numbers.
Provide complete arguments when asked to prove a statement in a question. You will be graded on how well you organize your proofs as well as the logical flow or your deductions.
Write your name on this page and staple it to your solutions.

**Due on Monday, February 22, 2010**

1. For a subset, $A$, of the real numbers and a real number, $c$, define the following sets

   (i) $A + c = \{y \in \mathbb{R} \mid y = x + c \text{ where } x \in A\}$, and

   (ii) $cA = \{y \in \mathbb{R} \mid y = cx \text{ where } x \in A\}$.

   Prove the following statements.

   (a) If $A$ is non–empty and bounded above, then
   $$\sup(A + c) = \sup A + c.$$

   (b) If $A$ is non–empty and bounded above and $c > 0$, then
   $$\sup(cA) = c \sup A.$$

   (c) What happens if $c < 0$ in part (b)? State and prove your result.

2. Let $A = \left\{ \dfrac{n+1}{n} \;\middle|\; n \in \mathbb{N} \right\}$.

   Prove that $A$ is bounded above and below and compute $\inf A$ and $\sup A$.

   Justify your calculations and prove any assertion you make.