Exam 1 (Part I)

Friday, February 19, 2010

Name: ______________________________

Provide complete arguments when asked to prove a statement in a question. You will be graded on how well you organize your proofs as well as the logical flow or your deductions. In this exam you'll be allowed to use the handout on the axioms of $\mathbb{R}$ (Handout #2). Use your own paper and/or the paper provided for you. Write you name on this page and staple it to your solutions. You have 50 minutes to work on the following 3 problems. Relax.

1. Provide concise answers to the following questions:

   (a) A subset, A, of the real numbers is said to be **bounded** if there exists a positive real number, M, such that

   $$|a| \leqslant M \quad \text{for all } \ a \in A.$$

   Give the negation of the statement

   "A is bounded."

   (b) Let A denote a subset of the real numbers and β a positive real number. Give the contrapositive for the following implication:

   $$t \in A \Rightarrow t \leq s - \beta.$$

2. Use the field and order axioms of the real numbers to prove the following.

   (a) Let $a, b \in R$. If $ab = 0$, then either $a = 0$ or $b = 0$.

   (b) Let $p \in \mathbb{R}$. If $p > 1$, then $p < p^2$.

3. Use the completeness axiom of $\mathbb{R}$ to prove that the set of natural numbers is not bounded above. Deduce, therefore, that for any real number, x, there exists a natural number, n, such that

   $$x < n.$$