Exam 2 (Part I)

Friday, April 2, 2010

Name: ________________________________

Provide complete arguments when asked to prove a statement in a question. You will be graded on how well you organize your proofs as well as the logical flow or your deductions. This is a closed–book, closed–notes exam. Use your own paper and/or the paper provided for you. Write you name on this page and staple it to your solutions. You have 50 minutes to work on the following 3 problems. Relax.

1. Let (x_n) denote a sequence of real numbers.

   (a) State precisely what the statement "(x_n) converges" means.

   (b) Let $x_n = \dfrac{1}{\sqrt{n}}$ for all $n \in \mathbb{N}$. Use the definition that you stated in the previous part to prove that (x_n) converges.

2. Let (x_n) denote a sequence of real numbers.

   (a) State precisely what it means for (x_n) to be a Cauchy sequence.

   (b) Prove that if (x_n) converges, the it is a Cauchy sequence.

3. Let $B \subseteq \mathbb{R}$ be a non–empty subset which is bounded below and put $\ell = \inf B$.

   (a) Prove that there exists a sequence of numbers in B which converges to ℓ.

   (b) Apply the result of the previous part to the set

   $$B = \{q \in \mathbb{Q} \mid q > 0 \text{ and } q^2 > 2\}$$

   to deduce that there exists a sequence of rational numbers $\{q_n\}$ which converges to $\sqrt{2}$.

   Note: You will need to prove that $\inf B = \sqrt{2}$.