Assignment #5

Due on Monday, February 15, 2010

Read Section 1.3 on *Analyzing Nonlinear Models,* pp. 20–28, in Allman and Rhodes.

Do the following problems

1. Suppose that X_t satisfies the *difference inequlity*

$$|X_{t+1}| \leq \eta |X_t| \quad \text{for} \quad t = 0, 1, 2, 3, \ldots$$

where $0 < \eta < 1$. Prove that

$$\lim_{t\to\infty} X_t = 0.$$

2. The *Principle of Linearized Stability* for the difference equation

$$N_{t+1} = f(N_t)$$

states that, if f is differentiable at a fixed point N^* and

$$|f'(N^*)| < 1,$$

then N^* is an assymptotically stable equilibrium solution.

In this problem we use the Principle of Linearized stability to analyze the following population model:

$$N_{t+1} = \frac{kN_t}{b + N_t}$$

where k and b are postive parameters.

(a) Write the model in the form $N_{t+1} = f(N_t)$ and give the fixed points of f. What conditions of k and b must be imposed in order to ensure that the model will have a non–negative steady state?

(b) Determine the stability of the equilibrium points found in part (a).

3. Problems 1.3.6 (d)(e) on page 29 in Allman and Rhodes.

4. Problems 1.3.7 (d)(e) on page 29 in Allman and Rhodes.

5. Problems 1.3.111 (a)(b)(c)(d) on page 30 in Allman and Rhodes.

Note: The code for the MATLAB® program `onepop` *may be downloaded from the courses website at* `http://pages.pomona.edu/~ajr04747`.