# Exam 2 (Part I)

Monday, April 5, 2010

Name: ______________________________

Show all significant work and justify all your answers. This is a closed book exam. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 2 problems. Relax.

1. Suppose that the rate at which a drug leaves the bloodstream and passes into the urine at a given time is proportional to the quantity of the drug in the blood at that time.

   (a) Write down and solve a differential equation for the quantity, $Q = Q(t)$, of the drug in the blood at time, $t$, in hours. State all the assumptions you make and define all the parameters that you introduce.

   (b) Suppose that an initial dose of $Q_o$ is injected directly into the blood, and that 20% of that initial amount is is left in the blood after 3 hours. Based on the solution you found in the previous part, write down $Q(t)$ for this situation and sketch its graph.

   (c) How much of the drug is left in the patient's body after 6 hours if the patient is given 100 mg initially?

2. Suppose a bacterial colony has $N_o$ bacteria at time $t = 0$. Let $M(t)$ denote the number of bacteria that develop certain mutation during the time interval $[0, t]$. Assume that, for small $\Delta t > 0$,

$$M(t + \Delta t) - M(t) \cong a\ (\Delta t)\ N(t), \tag{1}$$

   where $a$ is a positive constant, and $N(t)$ is the number of bacteria in the colony at time $t$.

   (a) Give an interpretation to what the expression in (1) is saying. In particular, provide a meaning for the constant, $a$, known as the *mutation rate.*

   (b) Let $\mu(t) = E(M(t))$ denote the expected value of the number of mutations in the time interval $[0, t]$. It is possible to prove, using the expression in (1), that $\mu = \mu(t)$ is differentiable and satisfies the differential equation

$$\frac{d\mu}{dt} = aN(t). \tag{2}$$

   Solve the differential equation in (2) assuming that $N(t)$ grows in time according to a Malthusian model with per–capita growth rate $k$, and that there are no mutant bacteria at time $t = 0$.