Assignment #9

Due on Monday, February 28, 2011

Read Section 3.3 on *Linear Approximation and Differentiability,* pp. 113–123, in Baxandall and Liebek's text.

Read Section 4.3 on *Differentiable Scalar Fields* in the class Lecture Notes (pp. 44–49).

Do the following problems

1. Let $U = \mathbb{R}^n \backslash \{\mathbf{0}\} = \{v \in \mathbb{R}^n \mid v \neq \mathbf{0}\}$ and define $f\colon \mathbb{R}^n \to \mathbb{R}$ by $f(v) = \|v\|$ for all $v \in \mathbb{R}$.

   (a) Prove that f is differentiable on U.

   (b) Prove that f is not differentiable at the origin in $\mathbb{R}^n$.

2. Let $f\colon \mathbb{R}^3 \to \mathbb{R}$ be give by $f(x, y, z) = x^2y + y^2z + z^2x$, for all $(x, y, z) \in \mathbb{R}^3$. Compute all the first partial derivatives of f and verify that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = 3f.$$

3. Find the gradient of f for each of the following scalar fields:

   (a) $f(x, y, z) = xe^{yz}$,

   (b) $f(x, y, z) = 1/\sqrt{x^2 + y^2 + z^2}$, $(x, y, z) \neq (0, 0, 0)$.

4. Let $f(x, y) = \begin{cases} (x^2 + y^2)\sin\left(\dfrac{1}{x^2 + y^2}\right), & \text{if } (x, y) \neq (0, 0); \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$

   (a) Show that the partial derivatives of f with respect to x and y do exist at $(0, 0)$, and compute $\dfrac{\partial f}{\partial x}(0, 0)$ and $\dfrac{\partial f}{\partial y}(0, 0)$.

   (b) Show that the partial derivatives of f with respect to x and y are not continuous at $(0, 0)$.

5. Let f be as in the previous problem. Show that f is differentiable at $(0, 0)$, and compute $Df(0, 0)$.