Exam 1

March 4, 2011

Name: ________________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. The points $P(1,0,0)$, $Q(0,1,0)$ and $R(0,0,1)$ determine a unique plane in three dimensional Euclidean space, $\mathbb{R}^3$.

   (a) Give the equation of the plane determined by P, Q and R.

   (b) Give the parametric equations of the line through the point $(0,0,0)$ which is orthogonal to the plane determined by P, Q and R.

   (c) Find the intersection between the line found in part (b) above and the plane determined by P, Q and R.

   (d) Find the (shortest) distance from the point $(0,0,0)$ to the plane determined by P, Q and R, and give the coordinates of the point in the plane which is closest to the origin. Justify your answers

2. Let U denote an open subset of $\mathbb{R}^n$, and let $F\colon U \to \mathbb{R}^m$ be a vector valued function defined on U.

   (a) State precisely what it means for F to be continuous at $u \in U$.

   (b) Let $\widehat{u}$ denote a unit vector in $\mathbb{R}^n$ and define $F\colon \mathbb{R}^n \to \mathbb{R}^n$ by $F(v) = P_{\widehat{u}}(v)$, the orthogonal projection of v onto the direction of $\widehat{u}$, for all $v \in \mathbb{R}^n$. Prove that F is continuous on $\mathbb{R}^n$

3. Let U denote an open subset of $\mathbb{R}^n$, and let $f\colon U \to \mathbb{R}$ be a scalar field defined on U.

   (a) Define what it means for f to be differentiable at $u \in U$.

   (b) Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be given by $f(x,y) = \sqrt{x^2+y^2}$ for all $(x,y) \in \mathbb{R}^2$. Show that f is not differentiable at $(0,0)$.

   (c) Let $U = \{(x,y) \in \mathbb{R}^2 \mid (x,y) \neq (0,0)\}$ and $f\colon U \to \mathbb{R}$ be given by

   $$f(x,y) = \sqrt{x^2+y^2}, \quad \text{for all } (x,y) \in U.$$

   Show that f is differentiable on U.
   Give $Df(x,y)$ for all $(x,y) \in U$, and compute the gradient of f in U.