Topics for Final Exam

1. **Euclidean Space**

   1.1 Definition of n–Dimensional Euclidean Space

   1.2 Spans, Lines and Planes

   1.3 Dot Product and Euclidean Norm

   1.4 Orthogonality and Projections

   1.5 The Cross Product in $\mathbf{R}^3$

2. **Continuous Functions**

   2.1 Vector fields, scalar fields and paths

   2.2 Definition of continuous function

   2.3 Compositions of Continuous Functions

   2.4 Limits and continuity

3. **Differentiability**

   3.1 Definition of differentiability

   3.2 The derivative as a linear approximation

   3.3 Derivatives of vector valued functions

   3.4 Derivatives of scalar fields

      i. The gradient

      ii. Partial derivatives

      iii. Directional derivatives

   3.5 The Jacobian matrix of a differentiable function

   3.6 The derivative of a composition of functions: The Chain Rule

4. **Differentiable Paths**

   4.1 C^1 curves and parametrizations

   4.2 Simple C^1 curves.

   4.3 Piecewise C^1 simple curves

   4.4 Simple closed curves

   4.5 Arclength of a curve; arclength parameter

5. **Integrals on Curves**

   5.1 Re-parametrizations

5.2 Path integrals

5.3 Line integrals

5.4 Flux across a closed curve

6. **Differential forms and the Fundamental Theorem of Calculus**

6.1 Differential 1–forms

6.2 Differential 0–forms

6.3 The differential of a 0–form

6.4 Differential 2–forms

6.5 Wedge product of 1–forms

6.6 Algebra of forms

6.7 The differential of a 1–form

6.8 Calculus of forms

6.9 Integration of forms

6.10 The Fundamental Theorem of Calculus

6.11 Evaluating differential 2–forms in oriented triangles: Double integrals

Relevant sections in the text: Sections 1.2, 2.1, 2.3, 2.5, 2.6, 2.7, 3.3, 3.6, 3.7, 3.8, 4.1, 4.2, 4.3, 4.4, 5.2, 5.3, 5.4, 5.5, 11.2 and 11.3.

Relevant chapters in the online class notes: Chapters 2, 3, 4 and 5.

Important Concepts: Euclidean space, dot product, orthogonal projections, cross product, continuous function, differentiability, the derivative map, partial derivatives, and the gradient, C^1 curves, piecewise C^1 curves, simple curves, simple closed curves, parametrizations, re–parametrizations, arclength, path integral, line integral, flux, differential forms, wedge product of forms, and double integrals.

Important Skills: Know how to compute projections; know how to find equations of lines and planes; know how to show that a function is continuous or not; know how to show whether a function is differentiable or not; know how to compute partial derivatives, gradients and directional derivative of scalar fields; Know how to compute the Jacobian matrix of a differentiable map, know how to apply the Chain Rule, know how to evaluate the arclength of C^1 curves, know how to evaluate path integrals, know how to evaluate line integrals, know how to compute flux across a simple closed curve, know how to evaluate differential 1–forms and differential 2–forms, know how to evaluate double integrals, know how to apply the Fundamental Theorem of Calculus.