Assignment #11

Due on Wednesday, February 29, 2012

Read Section 4.1 on *The Expectation of a Random Variable* in DeGroot and Schervish.

Do the following problems

1. An experiment consists of tossing a fair coin twice in a row. Let E_1 denote the event that a head comes up on the first toss, E_2 denote the event of heads on the second toss, and E_3 denote the event that exactly one head is thrown. Verify that E_1, E_2 and E_3 are pairwise independent but that

$$\Pr(E_3 \mid E_1 \cap E_2) \neq \Pr(E_3).$$

   Conclude therefore that the events E_1, E_2 and E_3 are not mutually independent.

2. Let X have the pdf $f_X(x) = 3x^2$ for $0 < x < 1$, zero elsewhere. Consider a random rectangle whose sides are X and $1 - X$. Determine the expected value of the area of the rectangle.

3. In a class of 50 students, the number of students n_i in each age class i is shown in the following Table 1.

Age i	n_i
18	20
19	22
20	4
21	3
25	1

Table 1: Number of students in each age class

   If a student is to be selected at random from the class, what is the expected value of the student's age?

4. Suppose that one word is to be selected at random from the sentence

   THE GIRL PUT ON HER BEAUTIFUL RED HAT.

   If X denotes the number of letters in the word that is selected, what is the value of $E(X)$?

5. Suppose that one letter is to be selected at random from the 30 letters in the sentence given in the previous exercise. If Y denotes the number of letters in the word in which the selected letter appears, what is the value of $E(Y)$?