Assignment #7

Due on Friday, February 10, 2012

Read Section 3.1 on *Random Variables and Discrete Distributions* in DeGroot and Schervish.

Do the following problems

1. For each of the following, find the value of the constant c for which the given function, $p(x)$, is the probability mass function (pmf) of some discrete random variable.

   (a) $p(x) = c \left(\dfrac{2}{3} \right)^x$, for $x = 1, 2, 3, \ldots$ and zero elsewhere.

   (b) $p(x) = cx$ for $x = 1, 2, 3, 4, 5$, and zero otherwise.

2. Suppose that two balanced dice are rolled, and let X denote the absolute value of the difference between the two numbers that appear. Determine and sketch the pmf of X.

3. Suppose that a box contains seven red balls and three blue balls. If five of them are selected at random, without replacement, determine the pmf of the number of red balls that will be obtained.

4. A civil engineer is studying a left–turn lane that is long enough to hold 7 cars. Let X denote the number of cars left in the lane at the end of randomly chosen red light. The engineer believes that the probability that $X = x$ is proportional to $(x + 1)(8 - x)$ for $x = 0, 1, \ldots, 7$ (the possible values of X).

   (a) Find the pmf for X.
   (b) Find the probability that X will be at least 5.

5. Select five cards at random and without replacement from an ordinary deck of playing cards. Let X denote the number of hearts in the five cards.

   (a) Find the probability mass function (pmf) of X. Denote it by $p(x)$.
   (b) Determine $\Pr(X \leqslant 1)$.
   (c) Find the cumulative distribution function, $F(x) = \Pr(X \leqslant x)$, and sketch its graph along with that of $p(x)$.