Exam 1

Friday, February 17, 2012

Name: ________________________________

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 questions. Relax.

1. Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space.

   (a) Given events A and B in $\mathcal{B}$ with $\Pr(B) > 0$. Give the definition of the conditional probability of A given B; i.e., $\Pr(A \mid B)$.

   (b) Define what it means for $A, B \in \mathcal{B}$ to be stochastically independent.

   (c) Define what it means for $A, B \in \mathcal{B}$ to be mutually exclusive.

   (d) Given that A and B are independent events in $\mathcal{B}$ with $\Pr(A) = \dfrac{1}{4}$ and $\Pr(B) = \dfrac{3}{4}$, compute $\Pr(A \cup B)$.

2. A bowl contains 4 chips of the same size and shape. One and only one of these chips is red.

   (a) In an experiment, chips are drawn at random from the bowl, one at a time and without replacement, until the red chip is drawn. Let X denote the number of draws needed to get the red chip. Give the pmf, p_X, of X.

   (b) In another experiment, chips are drawn at random from the bowl one at a time, but this time with replacement, until the red chip is drawn. Let Y denote the number of draws needed to get the red chip. Compute p_Y.

3. Let $X \sim \text{Uniform}(0, 1)$.

   (a) Give the pdf, f_X, for X.

   (b) Define $Y = \dfrac{1}{X}$. Compute the cdf, F_Y, for Y in terms of the cdf for X.

   (c) Compute the pdf, f_Y, for Y.

   (d) Compute the probability that Y is at least 2 given that Y is at most 4.