Exam 2

Wednesday, March 28, 2012

Name: _____

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 4 problems. Relax.

1. Suppose that $X \sim \text{Normal}(0, 1)$ and define $Y = |X|$.

   (a) Compute the cumulative distribution function, F_Y, of Y in terms of F_X, and then differentiate it with respect to y in order to determine the probability density function, f_Y.

   (b) Compute the expected value, $E(Y)$, of Y.

2. For $\mu \in \mathbb{R}$, let X and Y be independent, Normal $\left(\mu, \frac{1}{2}\right)$ random variables. Put $Z = X - Y$.

   (a) Compute the moment generating function, $\psi_Z(t)$, of Z.

   (b) Use the mgf computed in part (a) to determine the distribution for Z. Give the pdf, f_Z.

   (c) Give the expected value, $E(Z)$, and variance, $\text{Var}(Z)$, of Z.

3. A random point, (X, Y), is distributed uniformly on the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$.

   (a) Give the joint pdf for X and Y.

   (b) Compute the marginal distributions of X and Y.

   (c) Are X and Y independent? Justify your answer.

4. Let X and Y be independent, Exponential(2) random variables.

   (a) Give the joint probability density function for (X, Y).

   (b) Compute $\Pr(Y < X)$.