Math 151. Rumbos Spring 2008 1

Exam 3

Friday, April 27, 2012

Name: ─────────────────────────────

This is a closed book exam. Show all significant work and justify all your answers. Use your own paper and/or the paper provided by the instructor. You have 50 minutes to work on the following 3 problems. Relax.

1. Let $X_1, X_2, X_3, \ldots$ denote independent, identically distributed Poisson random variables with mean 1. Define $Y_n = \sum_{k=1}^{n} X_k,$ for $n = 1, 2, 3, \ldots$

   (a) Compute the moment generating function of Y_n, and use it to determine the distribution of Y_n for $n = 1, 2, 3, \ldots$

   (b) Give the expected value and variance of Y_n.

   (c) Compute $\Pr(Y_n \geqslant 1)$.

   (d) Determine the limiting distribution of the random variables $\dfrac{Y_n - n}{\sqrt{n}}$ as $n \to \infty$. Justify your answer.

   (e) Use your result in part (d) to estimate $\Pr(Y_n > 50)$ for $n = 49$.

2. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size $n = 100$ from an exponential distribution with mean 2.

   (a) Compute the probability, p, that a given observation from the random sample has a value that is at least 6.

   (b) Let Y denote the number of observations in the sample that are at least 6. Give the distribution of Y.

   (c) Estimate the probability that at most 10 of the observations from the random sample have values that are at least 6. Justify your answer.

3. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a distribution with mean μ and variance σ^2. Let $\overline{X}_n$ denote the sample mean.

   (a) Use the Chebyshev inequality to estimate $\Pr[|\overline{X}_n - \mu| \geqslant \sigma]$.

   (b) How large should n be so that the probability that the sample mean will be within one standard deviation of the mean is at least 0.99?