

## Assignment #4

Due on Monday, February 6, 2012

Read Section 3.1 on *Modeling Traffic Flow* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>.

**Background**

Let  $f: I \rightarrow \mathbf{R}$  denote a continuous, real-valued function defined on an open interval,  $I$ , of the real line. In this problem set you will establish the following result:

**Proposition A.** Suppose that

$$\int_a^b f(x) dx = 0 \quad \text{for all intervals } [a, b] \subset I; \quad (1)$$

then,  $f(x) = 0$  for all  $x \in I$ .

**Do** the following problems.

1. Assume that  $f: I \rightarrow \mathbf{R}$  is continuous and that  $f(x_o) \neq 0$  for some  $x_o \in I$ . Use the definition of continuity at  $x_o$ , with  $\varepsilon = \frac{|f(x_o)|}{2}$ , to deduce that there exists  $\delta > 0$  such that  $[x_o - \delta, x_o + \delta] \subset I$  and

$$x \in [x_o - \delta, x_o + \delta] \Rightarrow f(x_o) - \frac{|f(x_o)|}{2} < f(x) < f(x_o) + \frac{|f(x_o)|}{2}. \quad (2)$$

2. Let  $f$ ,  $x_o$  and  $\delta$  be as in Problem 1. Use (2) to show that, if  $f(x_o) > 0$ , then

$$x \in [x_o - \delta, x_o + \delta] \Rightarrow f(x) > \frac{|f(x_o)|}{2}. \quad (3)$$

3. Let  $f$ ,  $x_o$  and  $\delta$  be as in Problem 1. Use (2) to show that, if  $f(x_o) < 0$ , then

$$x \in [x_o - \delta, x_o + \delta] \Rightarrow f(x) < -\frac{|f(x_o)|}{2}. \quad (4)$$

4. Let  $f$ ,  $x_o$  and  $\delta$  be as in Problem 1. Use the results in the previous problems in (3) and (4) to show that, if  $f(x_o) \neq 0$ , then either

$$\int_{x_o-\delta}^{x_o+\delta} f(x) dx > \delta|f(x_o)| > 0 \quad \text{or} \quad \int_{x_o-\delta}^{x_o+\delta} f(x) dx < -\delta|f(x_o)| < 0.$$

5. Prove Proposition A through an indirect argument; that is, assume that (1) holds true, but  $f(x_o) = 0$  for some  $x_o \in I$ , and derive a contradiction.