Assignment #8

Due on Friday, March 9, 2012

Read Section 4.1 on *Random Variables* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`.

Do the following problems.

1. Recall that the expected value of a discrete random variable, X, with a countable number of possible values

$$x_1, x_2, x_3, \ldots,$$

is defined to be the sum $E(X) = \sum_{k=1}^{\infty} x_k p_X(x_k)$, where p_X is the probability distribution of X.

Show that, if X and Y are independent discrete random variables, then

$$E(X+Y) = E(X) + E(Y). \tag{1}$$

The result in (1) is true in general; that is, it is true for any finite number of random variables regardless of whether the random variables are discrete, continuous, or independent. The proof of (1) for the general case is found in any probability textbook.

2. **Variance of a Distribution.** Given a random variable, X, with expected value $\mu = E(X)$, the variance of X, denoted by $\mathrm{Var}(X)$, is defined to be

$$\mathrm{Var}(X) = E([X-\mu]^2); \tag{2}$$

that is, $\mathrm{Var}(X)$ is the mean square deviation of X from its mean $\mu = E(X)$.

Use the result about expectations of sums of random variables mentioned in Problem 1 to derive the formula

$$\mathrm{Var}(X) = E(X^2) - \mu^2, \tag{3}$$

where $\mu = E(X)$.

3. Use the formula in (3) in order to compute the variance of $X \sim \mathrm{Bernoulli}(p)$.

4. Use the formula in (3) in order to compute the variance of $Y \sim \text{Binomial}(n, p)$.

5. Consider the following random experiment: Toss a die until you get a six on the top face; then stop. Let X denote the number of tosses you make until you stop.

   (a) Explain why X is a discrete random variable. What are the possible values for X?

   (b) Assume the probability of tossing a six is p, where $0 < p < 1$. Compute the probability distribution of X; that is, for each value k of X, compute

   $$p_X(k) = \Pr[X = k].$$

   (c) Compute the expected value of X.