Math 183. Mathematical Modeling
Pomona College
Spring 2012

Modeling Projects

1. **Modeling Mutations to Resistance**

In class we saw how to use a Poisson process to model the number of mutations in a bacterial colony as a function of time. In this project, you will extend the analysis to take into account the fact that the size of the bacterial colony is increasing in time. Part of the analysis was begun by Luria and Delbrück in their paper in *Genetics*, **28**:491-511, 1943, in the context of mutations that led to strains of *E. Coli* bacteria that were resistant to certain virus. In their 1943 paper, Luria and Delbrück did not compute the distribution. This was done later by Lea and Coulson (J. Genetics, **49**:264-285, 1949). Many years later, a very simple derivation of the distribution was obtained by Ma, Sandri and Sarkar (J. Applied Probability, **29**(2): 255-267, 1992).

The task for this project is to discuss the Luria- Delbrück distribution as a birth-and-mutation random process and to present the Ma, Sandri and Sarkar solution.

2. **Statistical Testing of Models**

The goal of this project is to test the two models presented in the Luria-Delbrück paper of 1943 (*Genetics*, **28**:491-511, 1943). One of the models predicts a Poisson process, while the other predicts the Luria- Delbrück distribution. You will analyze the data from the experimental results presented in the paper. You will have to do some research on techniques for testing whether data from experiments fit the predictions of theoretical models. You will also have to delve into parameter estimation.

3. **Shock Waves on the Highway**

In class and in the notes we derived a continuous, deterministic model for traffic on a long, straight, one-lane road. The model we presented models a traffic stream as a fluid and uses analysis for a one-dimension, compressible fluid to derive a first-order nonlinear partial differential equation. The original analysis goes back to Lighthill and Whitman in 1955 (*Proceedings Royal Society of London*, **229**:317-345, 1955) and Richards (Operations Research, **4**:42-51, 1956).

The task for this project is to present the analysis of Lighthill, Whitman and Ricarhds' model for traffic flow.

4. **Probabilistic Models of Traffic Flow**

Traffic flow is, perhaps, more adequately modeled by a discrete system of moving cars and some random variables whose averages are those that come up in the continuous and deterministic models of Lighthill, Whitman and Ricarhds'. Jiabaru and Liu present one of those stochastic models of traffic flow in *Transportation Research B*, **46**:156-174, 2012. In this project you'll present the probabilistic model in *Transportation Research B*, **46**:156-174, and discuss the various distributions used in the model. Pay close attention to the connections between the Jiabaru-Liu models and the Lighthill-Whitman-Richards one.

5. **A Queueing Model for Road Traffic Flow**

Another probabilistic approach to traffic flow modeling is to use queueing theory. In 1961, Allan J. Miller proposed one of those models in *Journal Royal Statistical Society B*, **23**(1):64-90, 1961. Your task is to explain the model and analysis proposed in that paper.

6**. Microbial Competition in a Chemostat**

At the start of the semester we looked at a continuous, deterministic model for a single-species bacterial population growing in a chemostat. In this project, you will look at the case of more than one species of microbes growing in a chemostat. Start out by reading the survey article by Hansen and Hubbell in *Science*, **207**:1491-1493, 1980. The general mathematic theory had been discussed previously by Hsu, Hubbell and Waltman in 1977 (*SIAM J. Applied Mathematics*, **32**(2):366-383, 1977). Your task is to present the model and some of the analysis. The more recent paper by Hsu and Waltman (*SIAM J. Applied Mathematics*, **52**(2):528-540, 1992) might also be helpful.