Assignment #1

Due on Wednesday, January 30, 2013

Read Section 2.2, *Bacterial Growth in a Chemostat,* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`.

Do the following problems

1. The diagram in Figure 1 shows a simplification of the chemostat model discussed in Section 2.2 in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`. The compartment in the diagram in Figure 1 represents culture chamber con-

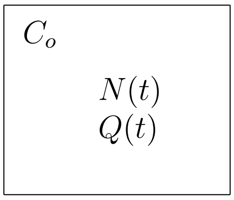

Figure 1: One–Compartment Model

taining $N(t)$ bacteria and a quantity $Q(t)$ of nutrient at time t. The quantities N and Q are assumed to be differentiable functions of t. Assume also that there is no flow of culture in or out of the chamber and that the culture in the chamber is kept well–stirred. In addition, assume that there is an initial amount of nutrient, Q_o, at an initial concentration of C_o, and that there are N_o bacteria at time $t = 0$. Postulate that the *per–capita* growth, $K(c)$, is a function of the nutrient concentration,
$$c(t) = \frac{Q(t)}{V},$$
where V is the volume of the culture, which is assumed to be constant. Assuming that $Y = 1/\alpha$ new cells are produced as s result of consumption of one unit of nutrient, apply conservations principles obtain a model for the evolution of N and Q in the chamber.

2. Combine the differential equations derived in Problem 1 to show that
$$\frac{d}{dt}[\alpha N + Q] = 0.$$
Deduce therefore that
$$\alpha N(t) + Q(t) = \alpha N_o + Q_o, \qquad \text{for all } t.$$

3. Denote $\alpha N_o + Q_o$ by A_o and use the result in Problems 2 to obtain the formula

$$c = \frac{A_o}{V} - \frac{\alpha}{V} N \tag{1}$$

for the concentration of nutrient.

   (a) Give and interpretation for the expression in (1).

   (b) Denote A_o/V by c_o. Explain why c_o is the nutrient concentration in the absence of bacteria.

4. Assume the constitutive equation $K(c) = mc$, where m is a positive constant of proportionality. Combine the results in Problems 1 and 3 to derive the differential equation

$$\frac{dN}{dt} = mN\left(c_o - \frac{\alpha}{V} N\right). \tag{2}$$

5. Set $r = mc_o$ and $L = \dfrac{c_o V}{\alpha}$, and use (2) to derive a well–know differential equation model for bacterial growth. What is the equation?