Exam 2

Due on Wednesday, April 17, 2013

Name: ________________________________

This is an open–notes exam; you may consult your own notes or the class notes in my courses website at `http://pages.pomona.edu/~ajr04747/`. Students are expected to work individually on these problems. You may not consult with anyone.

Show all significant work and provide reasoning for all your assertions.

Write your name on this page and staple it to your solutions. Turn in your solutions at the start of class on **Wednesday, April 17, 2013.**

I have read and agree to these instructions. Signature: ____________________

1. Imagine an experimental apparatus consisting of a very long cylindrical tube of cross–sectional area A. The tube is placed horizontally along the x–axis. Initially there is an impermeable membrane at $x = 0$. For $x < 0$, the tube if filled with a solutions containing a substance with concentration C_o (in number of particles per unit volume). For $x > 0$, the tube is filled with a solution in which the concentration of the substance is 0. At time $t = 0$, the membrane at $x = 0$ is removed and the substance on the left begins to disperse towards the right. At any x and time $t > 0$, let $C(x,t)$ denote the concentration of the substance at points on the cross–section at x and at time t. Assume that C is a C^2 function; that is, the second partial derivatives of C exist and are continuous for all x and all $t > 0$.

   (a) Write down a differential equation model that describes the evolution of $C(x,t)$ in time, subject to the initial condition described above.

   (b) Give a solution to the initial value problem formulated in part (a). Express the solution in terms of the Error function.

   (c) Use Fick's first law of diffusion to compute the flux, $J_x(x,t)$, of the substance in the solution. Give an interpretation for $J_x(0,t)$.

   (d) Let M_t denote the number of particles of the substance in the solution that crossed the cross–section at $x = 0$ during the time interval $[0,t]$. Use the result from part (c) to derive the formula

   $$M_t = \frac{AC_o\sqrt{D}}{\sqrt{\pi}}\ \sqrt{t}, \qquad (1)$$

   where D is the diffusivity of the medium in the tube.

(e) Give an interpretation for (1) and explain how you can use the result in (1) to estimate the diffusivity of the medium.

2. *(Estimating the Diffusivity)* Suppose that the tube described in Problem 1 has total length L and that the equation in (1) holds true in this case as well. Assume also that the middle cross–section of the tube is located at $x = 0$.

   (a) Let M_o denote the number of particles to the left of $x = 0$ in the setup described in Problem 1 before the membrane at $x = 0$ is removed. Give a formula for computing M_o.

   (b) Use your result from part (a) and the formula in (1) to derive the formula

$$\frac{M_t}{M_o} = 2\frac{\sqrt{D}}{\sqrt{\pi L^2}}\,\sqrt{t}. \qquad (2)$$

   (c) The graph in Figure 1 on page 3 of this exam shows a plot of the ratio M_t/M_o versus $\sqrt{t}$ based on data collected in experiments[1] involving diffusion of bromophenol blue anions (series a) and KCL (series b). The lines in the plot in Figure 1 are the least–square regression lines. The length of the tube, L, in the bromophenol blue experiment is 8.7 cm and that in the KCl experiment is 9.3 cm. The time is measured in seconds.

   Use (2) and the data in Figure 1 to estimate the diffusion coefficients for (i) bromophenol blue, and (ii) KCl.

[1] Crooks, J. E., *Measurement of Diffusion Coefficients. Journal of Chemical Education,* 1989, **66**, pp. 614–615

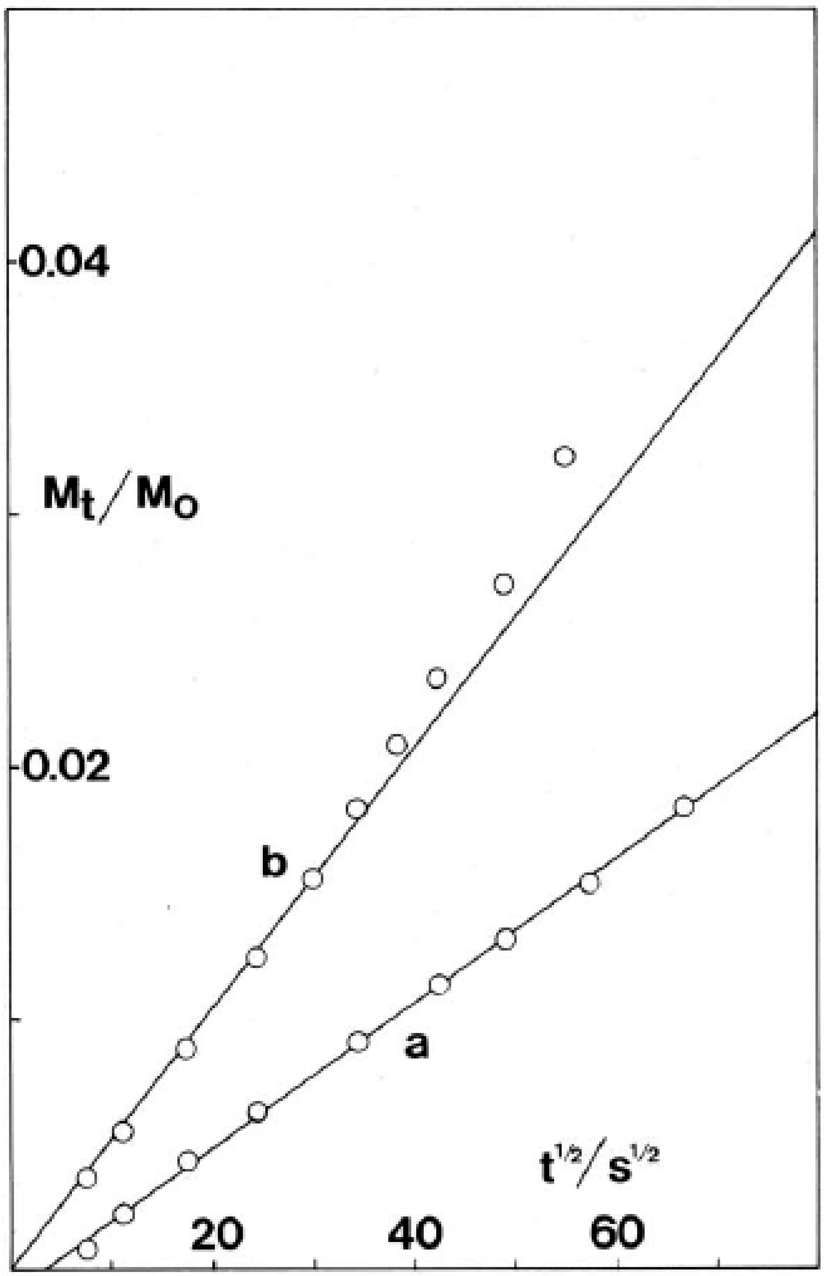

Figure 1: M_t/M_o versus $\sqrt{t}$. Series a: bromophenol blue anion. Series b: KCl