Assignment #15

Due on Wednesday, March 12, 2014

Read Section 4.3 on *Moments* and Section 4.4 on *Variance* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Section 4.3 on *Variance* and Section 4.4 on *Moments* in DeGroot and Schervish.

Do the following problems

1. Let X be a random variable with moment generating function (mgf) ψ_X.

   (a) Let $Y = cX$, where c is a constant. Compute the mgf of Y in terms of ψ_X.

   (b) Let $Y = X + a$, where a is a constant. Compute the mgf of Y in terms of ψ_X.

2. Let X be a random variable with moment generating function (mgf) ψ_X, expected value μ and variance σ^2. Put $Y = \dfrac{X - \mu}{\sigma}$

   (a) Compute the mgf of Y in terms of ψ_X.

   (b) Use the moment generating function found in part (a) to compute $E(Y)$ and $\text{Var}(Y)$.

3. Let X have mgf

$$\psi_X(t) = (1 - p)e^{-t} + pe^t, \quad \text{for all } t \in \mathbb{R},$$

   where $0 < p < 1$.

   (a) Give the distribution of X.

   (b) Use the mgf to find $E(X)$ and $\text{Var}(X)$.

4. Let X have mgf

$$\psi_X(t) = \frac{1}{6}e^{-2t} + \frac{1}{3}e^{-t} + \frac{1}{2}e^t, \quad \text{for all } t \in \mathbb{R}.$$

   Compute $\Pr(|X| \leqslant 1)$.

5. Suppose that X is a nonnegative random variable and that $\psi_X(\delta) < \infty$ for some $\delta > 0$. Show that $\psi_X(t)$ exists for all $t \in [0, \delta]$.