Assignment #19

Due on Wednesday, April 9, 2014

Read Section 6.2 on *The Poisson Distribution* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 5.4 on *The Poisson Distribution* in DeGroot and Schervish.

Do the following problems

1. We have seen in the lecture that if X has a Poisson distribution with parameter $\lambda > 0$, then it has the pmf:

$$p_X(k) = \frac{\lambda^k}{k!}\, e^{-\lambda} \quad \text{for } k = 0, 1, 2, 3, \ldots; \text{ zero elsewhere.}$$

   Use the fact that the power series $\displaystyle\sum_{m=0}^{\infty} \frac{x^m}{m!}$ converges to e^x for all real values of x to compute the mgf of X.

   Use the mgf of X to determine the mean and variance of X.

2. Let $X_1,\ X_2,\ \ldots\ X_m$ be independent random variables satisfying $X_i \sim \text{Poisson}(\lambda)$ for all $i = 1, 2, \ldots, m$ and some $\lambda > 0$. Define

$$Y = X_1 + X_2 + \cdots + X_m.$$

   Determine the distribution of Y; that is, compute its pmf.

3. Suppose that on a given weekend the number of accidents at a certain intersection has a Poisson distribution with mean 0.7. What is the probability that there will be at least three accidents in the intersection during the weekend?

4. Suppose that a certain type of magnetic tape contains, on average, three defects per 1000 feet. What is the probability that a roll of tape 1200 feet long contains no defects?

5. Suppose that X_1 and X_2 are independent random variables and that X_i has a Poisson distribution with mean λ_i ($i = 1, 2$). For a fixed value of k ($k = 0, 1, 2, 3, \ldots$), determine the conditional distribution of X_1 given that $X_1 + X_2 = k$.