Assignment #27

Due on Monday, April 28, 2014

Read Chapter 8 on *Introduction to Estimation* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 7.2 on *The Chi–Square Distribution* in DeGroot and Schervish.

Do the following problems

1. Let $X_1, X_2, \ldots, X_n$ denote a random sample from a Normal(μ, σ^2) distribution. Define
$$Y_n = \frac{1}{\sigma^2}\sum_{k=1}^{n}(X_k - \mu)^2,$$
for each $n = 1, 2, 3, \ldots$. Determine the distribution of Y_n.

2. Let (Z_k) denote a sequence of iid Norma$(0, 1)$ random variables and define
$$Y_n = \sum_{k=1}^{n} Z_k^2, \qquad \text{for } n = 1, 2, 3, \ldots$$
Give the limiting distribution of $\dfrac{Y_n - n}{\sqrt{2n}}$ as n tends to infinity.

3. Let (Z_k) denote a sequence of iid Norma$(0, 1)$ random variables and define $Y_n = \displaystyle\sum_{k=1}^{n} Z_k^2$, for $n = 1, 2, 3, \ldots$. show that $\dfrac{Y_n}{n}$ converges to 1 in probability.

4. Let $X \sim \chi^2(n)$ and $Y \sim \chi^2(m)$ be independent random variables. Define $W = X + Y$ and determine the distribution of W.

5. Let X and Y denote independent, Normal$(0, \sigma^2)$ random variables. Estimate the probability
$$\Pr\left(X^2 + Y^2 > \frac{\sigma^2}{2}\right).$$