

## Assignment #3

Due on Monday, February 3, 2014

**Read** Sections 2.4 on *Defining a Probability Function* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

**Read** Section 1.5 on *The Definition of Probability* in DeGroot and Schervish.

**Do** the following problems

1. Consider two events  $A$  and  $B$  such that  $\Pr(A) = 1/3$  and  $\Pr(B) = 1/2$ . Determine the value of  $\Pr(B \cap A^c)$  for each of the following conditions:
  - (a)  $A$  and  $B$  are disjoint;
  - (b)  $A \subseteq B$ ;
  - (c)  $\Pr(A \cap B) = 1/8$ .

2. Consider two events  $A$  and  $B$  with  $\Pr(A) = 0.4$  and  $\Pr(B) = 0.7$ . Determine the maximum and minimum possible values for  $\Pr(A \cap B)$  and the conditions under which each of these values is attained.

3. Prove that for every two events  $A$  and  $B$ , the probability that exactly one of the two events will occur is given by the expression

$$\Pr(A) + \Pr(B) - 2\Pr(A \cap B).$$

4. Let  $A$  and  $B$  be elements in a  $\sigma$ -field  $\mathcal{B}$  on a sample space  $\mathcal{C}$ , and let  $\Pr$  denote a probability function defined on  $\mathcal{B}$ . Recall that  $A \setminus B = \{x \in A \mid x \notin B\}$ . Prove that if  $B \subseteq A$ , then

$$\Pr(A \setminus B) = \Pr(A) - \Pr(B).$$

5. Let  $(\mathcal{C}, \mathcal{B}, \Pr)$  denote a probability space, and  $B$  an event in  $\mathcal{B}$  with  $\Pr(B) > 0$ . Let

$$\mathcal{B}_B = \{D \subset \mathcal{C} \mid D = E \cap B \text{ for some } E \in \mathcal{B}\}.$$

We have already seen that  $\mathcal{B}_B$  is a  $\sigma$ -field.

Let  $P_B: \mathcal{B}_B \rightarrow \mathbb{R}$  be defined by  $P_B(A) = \frac{\Pr(A)}{\Pr(B)}$  for all  $A \in \mathcal{B}_B$ . Verify that  $(B, \mathcal{B}_B, P_B)$  is a probability space; that is, show that  $P_B: \mathcal{B}_B \rightarrow \mathbb{R}$  is a probability function.