Math 151. Probability–Rumbos **Spring 2014**

Review Problems for Exam 1

(1) There are 5 red chips and 3 blue chips in a bowl. The red chips are numbered $1, 2, 3, 4, 5$ respectively, and the blue chips are numbered $1, 2, 3$ respectively. If two chips are to be drawn at random and without replacement, find the probability that these chips are have either the same number or the same color.

(2) A person has purchased 10 of 1,000 tickets sold in a certain raffle. to determine the five prize winners, 5 tickets are drawn at random and without replacement. Compute the probability that this person will win at least one prize.

(3) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let E_1, E_2 and E_3 be mutually disjoint events in $\mathcal{B}$. Find $\Pr[(E_1 \cup E_2) \cap E_3]$ and $\Pr(E_1^c \cup E_2^c)$.

(4) Let $(\mathcal{C}, \rfloor\dashv\updownarrow\mathcal{B}, \Pr)$ denote a probability space, and let A and B events in $\mathcal{B}$. Show that

$$\Pr(A \cap B) \leq \Pr(A) \leq \Pr(A \cup B) \leq \Pr(A) + \Pr(B).$$

(5) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let E_1, E_2 and E_3 be mutually independent events in $\mathcal{B}$ with probabilities $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, respectively. Compute the exact value of $\Pr(E_1 \cup E_2 \cup E_3)$.

(6) Let $(\mathcal{C}, \mathcal{B}, \Pr)$ denote a probability space, and let E_1, E_2 and E_3 be mutually independent events in $\mathcal{B}$ with $\Pr(E_1) = \Pr(E_2) = \Pr(E_3) = \frac{1}{4}$. Compute $\Pr[(E_1^c \cap E_2^c) \cup E_3]$.

(7) A bowl contains 10 chips of the same size and shape. One and only one of these chips is red. Draw chips from the bowl at random, one at a time and without replacement, until the red chip is drawn. Let X denote the number of draws needed to get the red chip.

(8) Let X have pmf given by $p_X(x) = \frac{1}{3}$ for $x = 1, 2, 3$ and $p(x) = 0$ elsewhere. Give the pmf of $Y = 2X + 1$.

(9) Let X have pmf given by $p_X(x) = \left(\frac{1}{2}\right)^x$ for $x = 1, 2, 3, \ldots$ and $p_X(x) = 0$ elsewhere. Give the pmf of $Y = X^3$.

(10) Let $f_X(x) = \begin{cases} \dfrac{1}{x^2} & \text{if } 1 < x < \infty; \\ 0 & \text{if } x \leq 1, \end{cases}$ be the pdf of a random variable X. If E_1 denote the interval $(1, 2)$ and E_2 the interval $(4, 5)$, compute $\Pr(E_1)$, $\Pr(E_2)$, $\Pr(E_1 \cup E_2)$ and $\Pr(E_1 \cap E_2)$.

(11) A *mode* of a distribution of a random variable X is a value of x that maximizes the pdf or the pmf. If there is only one such value, it is called *the mode of the distribution*. Find the mode for each of the following distributions:

(a) $p(x) = \left(\dfrac{1}{2}\right)^x$ for $x = 1, 2, 3, \ldots$, and $p(x) = 0$

(b) $f(x) = \begin{cases} 12x^2(1 - x), & \text{if } 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}$

(12) Let X have pdf $f_X(x) = \begin{cases} 2x, & \text{if } 0 < x < 1; \\ 0, & \text{elsewhere.} \end{cases}$
Compute the probability that X is at least $3/4$, given that X is at least $1/2$.

(13) Divide a segment at random into two parts. Find the probability that the largest segment is at least three times the shorter.

(14) Let X have pdf $f_X(x) = \begin{cases} x^2/9, & \text{if } 0 < x < 3; \\ 0, & \text{elsewhere.} \end{cases}$
Find the pdf of $Y = X^3$.