Exam 2 (Part II)

Due on Monday, March 31, 2014

Name: ______________________________

This is the out–of–class portion of Exam 2. There are three questions in this portion of the exam. This is a closed–book and closed–notes exam; you may consult only the "Special Distributions" handout. You may work on these questions as long as you wish. Show all significant work and give reasons for all your answers.

Students are expected to work individually on these problems. You may not consult with anyone.

Please, write your name on this page and staple it to your solutions. Turn in your solutions at the start of class on **Monday, March 31, 2014.**

I have read and agree to these instructions. Signature: ______________________

1. A device contains two circuits. The second circuit is a backup for the first, so the second is used only when the first has failed. The device fails when and only when the second circuit fails.

   Let X and Y be the times at which the first and second circuits fail, respectively. Assume that X and Y have joint probability density function

$$f_{(X,Y)}(x,y) = \begin{cases} ce^{-x-2y}, & \text{for } 0 < x < y < \infty; \\ 0, & \text{elsewhere.} \end{cases} \tag{1}$$

   (a) Determine the value of c in the defintion of $f_{(X,Y)}$ in (1).

   (b) What is the expected time at which the device fails?

2. In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that $p_{n+1} = \frac{1}{5}p_n$, for $n = 0, 1, 2, 3, \ldots$, where p_n represents the probability that the policyholder files n claims during the three-year period.

   (a) Determine the value of p_o and give the pmf for N, the number of claims a policyholder files in the three–year period.

   (b) Compute the expected number of claims filed by a policyholder in the three–year period.

3. Let X_1 and X_2 be independent Bernoulli(p) random variables, where $0 < p < 1$. Define $X = X_1 + X_2$ and $Y = 2X_1$.

   (a) Compute the mean and variance of X.

   (b) Compute the mean and variance of Y.

   (c) Are the variances of X and Y the same? If not, give an intuitive explanation for the difference.

   (d) Compute the joint pmf of (X, Y) and list its values by completing the table below.

$X\backslash Y$	0	2
0		
1		
2		

   (e) Compute the marginal distributions of X and Y and determine whether or not X and Y are independent. Explain your reasoning.