

Exam 3 (Part I)

Friday, May 2, 2014

Name: _____

This is the in-class portion of Exam 3. This is a closed-book and closed-notes exam; you may consult only the “Special Distributions” and the “Normal Distribution Probabilities Table” handouts.

Show all significant work and give reasons for all your answers. Use your own paper and/or the paper provided by the instructor. You have up to 50 minutes to work on the following 2 questions. Relax.

1. Let X and Y denote independent random variables such that $X \sim \text{Normal}(\mu, 0.5)$ and $Y \sim \text{Normal}(-\mu, 0.5)$, for some real parameter μ .
 - (a) Give the distribution of $X + Y$. Explain the reasoning leading to your answer.
 - (b) Give the distribution of $(X + Y)^2$. Explain the reasoning leading to your answer.
 - (c) Estimate the probability $\Pr[(X + Y)^2 < 0.25]$. Explain the reasoning leading to your answer.

2. Let X_1, X_2, \dots, X_n denote a random sample from a distribution with mean μ and variance σ^2 , and let \bar{X}_n denote the sample mean.
 - (a) State what the Central Limit Theorem says about the limiting distribution of \bar{X}_n as n increases to infinity.
 - (b) State what the Weak Law of Large Numbers says about the limit of \bar{X}_n as $n \rightarrow \infty$.
 - (c) How large should n be so that the probability that \bar{X}_n is within one standard deviation of μ is at least 99%. Explain the reasoning leading to your answer.