

Assignment #1

Due on Wednesday, January 28, 2015

Read Chapter 1 in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Chapter 2 on *Introduction to Modeling* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Do the following problems

1. The diagram in Figure 1 shows a simplification of the chemostat model discussed in Section 2.2 in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>. The compartment in the diagram in Figure 1 represents a culture chamber con-

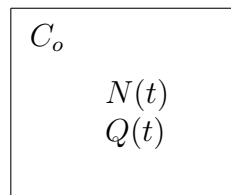


Figure 1: One-Compartment Model

taining $N(t)$ bacteria and a quantity $Q(t)$ of nutrient at time t . The quantities N and Q are assumed to be differentiable functions of t . Assume also that there is no flow of culture in or out of the chamber and that the culture in the chamber is kept well-stirred. In addition, assume that there is an initial amount of nutrient, Q_o , at an initial concentration of C_o , and that there are N_o bacteria at time $t = 0$. Postulate that the *per-capita* growth, $K(c)$, is a function of the nutrient concentration,

$$c(t) = \frac{Q(t)}{V},$$

where V is the volume of the culture, which is assumed to be constant. Assuming that $Y = 1/\alpha$ new cells are produced as a result of consumption of one unit of nutrient, apply conservation principles to obtain a model for the evolution of N and Q in the chamber.

2. Combine the differential equations derived in Problem 1 to show that

$$\frac{d}{dt}[\alpha N + Q] = 0.$$

Deduce therefore that

$$\alpha N(t) + Q(t) = \alpha N_o + Q_o, \quad \text{for all } t.$$

3. Denote $\alpha N_o + Q_o$ by A_o and use the result in Problems 2 to obtain the formula

$$c = \frac{A_o}{V} - \frac{\alpha}{V}N \tag{1}$$

for the concentration of nutrient.

- (a) Give an interpretation for the expression in (1).
(b) Denote A_o/V by c_o . Explain why c_o is the nutrient concentration in the absence of bacteria.
4. Assume the constitutive equation $K(c) = mc$, where m is a positive constant of proportionality. Combine the results in Problems 1 and 3 to derive the differential equation

$$\frac{dN}{dt} = mN \left(c_o - \frac{\alpha}{V}N \right). \tag{2}$$

5. Set $r = mc_o$ and $L = \frac{c_o V}{\alpha}$, and use (2) to derive a single equation describing the growth of bacteria in the chamber. This equation is known as the Logistic Equation.