

Assignment #10

Due on Friday, March 6, 2015

Read Section 4.2.1 on *Fundamental Matrices* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Chapter 3, on *Linear Systems*, in Blanchard, Devaney and Hall.

Background and Definitions.

Commuting Matrices. Given square matrices A and B of the same size, we say that A and B commute if

$$AB = BA.$$

Do the following problems

1. Let $J = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$. Verify that the corresponding fundamental matrix, E_J , commutes with J .
2. Let $J = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$. Verify that the corresponding fundamental matrix, E_J , commutes with J .
3. Let $J = \begin{pmatrix} \alpha & -\beta \\ \beta & \alpha \end{pmatrix}$. Verify that the corresponding fundamental matrix, E_J , commutes with J .
4. Let A denote a 2×2 matrix with real coefficients, and let E_A denote the fundamental matrix for the system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}.$$

Show that A and E_A commute.

5. Let E_A denote the fundamental matrix of the system

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

and put $Y(t) = [E_A(t)]^{-1}$, for all $t \in \mathbb{R}$. Show that Y is the fundamental matrix of the system

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = -A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$