Assignment #11

Due on Monday, March 9, 2015

Read Section 4.2.2 on *Existence and Uniqueness* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`

Read Chapter 3, on *Linear Systems,* in Blanchard, Devaney and Hall.

Do the following problems

1. Prove that the initial value problem for the second order ODE

$$\begin{cases} \dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + cx = 0; \\ \\ x(0) = x_o; \\ x'(0) = y_o, \end{cases}$$

   where b, c, x_o and y_o are given real constants, has unique solution that exists for all $t \in \mathbb{R}$.

2. Let b and c be given real constants. Suppose that $x_1 \colon \mathbb{R} \to \mathbb{R}$ is a solution to the IVP

$$\begin{cases} \dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + cx = 0; \\ \\ x(0) = 1; \\ x'(0) = 0, \end{cases}$$

   and $x_2 \colon \mathbb{R} \to \mathbb{R}$ be a solution to the IVP

$$\begin{cases} \dfrac{d^2x}{dt^2} + b\dfrac{dx}{dt} + cx = 0; \\ \\ x(0) = 0; \\ x'(0) = 1. \end{cases}$$

   Prove that x_1 and x_2 are linearly independent.

   Suggestion: Begin with the equation

$$c_1x_1(t) + c_2x_2(t) = 0, \qquad \text{for all } t \in \mathbb{R}.$$

3. Let b and c be given real constants. Let $x_1\colon \mathbb{R} \to \mathbb{R}$ and $x_2\colon \mathbb{R} \to \mathbb{R}$ be linearly independent solutions of the second order differential equation

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0. \tag{1}$$

Prove that any solution of (1) must be of the form

$$x(t) = c_1x_1(t) + c_2x_2(t), \quad \text{for all } t \in \mathbb{R}.$$

4. Let b and c be given real constants. Suppose that λ_1 and λ_2 be distinct real solutions to the equation

$$\lambda^2 + b\lambda + c = 0$$

Prove that the general solution of the equation

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + cx = 0$$

is given by

$$x(t) = c_1e^{\lambda_1 t} + c_2e^{\lambda_2 t}, \quad \text{for all } t \in \mathbb{R},$$

and arbitrary constants c_1 and c_2.

5. Find the solution to the initial value problem for the following second order differential equation:

$$\begin{cases} x'' - x = e^t; \\ x(0) = 1; \\ x'(0) = 0. \end{cases}$$