Assignment #16

Due on Friday, April 17, 2015

Read Section 6.1 on *Nondimensionalization,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 6.2, on *Analysis of one–dimensional systems,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

1. Consider the chemostat model without flow in or out of a single chamber of fixed volume V depicted in Figure 1, where c_o is the initial concentration of nutrient.

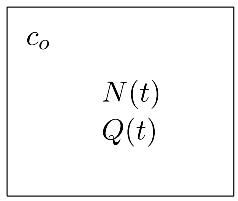

Figure 1: One–Compartment Chemostat Model

Proceed as in Problems 1–4 in Assignment #1, assuming this time that the *per capita* growth rate is given by the Michaelis-Menten enzyme kinetics relation

$$K(c) = \frac{rc}{a+c}, \tag{1}$$

where $c = Q/V$ is the nutrient concentration in the growth medium, to derive a differential equations satisfied by N and Q, where $Q(t)$ denotes the amount of nutrient in the chamber at time t.

You will need to use the yield $Y = 1/\alpha$, or the number of new cells produced in the chemostat due to consumption of one unit of nutrient.

Give an interpretation for the parameter r.

2. Use your result in Problem 1 to obtain a first–order ODE satisfied by N of the form

$$\frac{dN}{dt} = F(N; a, \beta, \gamma, r) \tag{2}$$

where the parameters β and γ are obtained as follows:

Let N_o and Q_o denote the initial population size and initial amount of nutrient in the chamber, respectively. Set $q_o = \alpha N_o + Q_o$; then set

$$\beta = \frac{\alpha}{V} \quad \text{and} \quad \gamma = \frac{q_o}{V}.$$

3. Introduce dimensionless variables

$$u = \frac{N}{\mu} \quad \text{and} \quad \tau = \frac{t}{\lambda} \tag{3}$$

to rewrite the equation (2) in the form

$$\frac{du}{d\tau} = \frac{u(\delta - u)}{1 - u}, \tag{4}$$

where δ is a dimensionless parameter.

Give formulas for the scaling parameters μ and λ in (3) and for the dimensionless parameter δ in (4).

Determine whether $\delta \geqslant 1$ or $0 < \delta < 1$.

4. Apply the principle of linearized stability (if applicable) to the ODE in (4) to determine whether or not the equilibrium points of the equation are stable or unstable.

5. Sketch possible solutions of the equation in (4).

Give an interpretation to what the solutions predict about the chemostat system modeled in Problem 1.