

Assignment #2

Due on Friday, January 30, 2015

Read Section 3.1 on *Types of Differential Equations* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Section 2.1, on *Modeling via Systems*, in Blanchard, Devaney and Hall.

Do the following problems

1. The second order differential equation

$$m \frac{d^2 s}{dt^2} = -ks - bv + f(t), \quad (1)$$

models a spring–mass system consisting of an object of mass m , attached to a spring with stiffness constant k , subject to a frictional force proportional to the velocity $v(t) = s'(t)$, for all t , that is also driven by a time–dependent force $f(t)$.

Express the system in (1) as a two dimensional system of first order differential equations.

2. Let $g: \mathbb{R} \rightarrow \mathbb{R}$ denote a continuous functions. Express the second order equation

$$x'' + g(x) = 0$$

as a system of first–order equations.

3. Given real numbers λ_1 and λ_2 , let

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} c_1 e^{\lambda_1 t} \\ c_2 e^{\lambda_2 t} \end{pmatrix}, \quad \text{for } t \in \mathbb{R}, \quad (2)$$

for constants c_1 and c_2 .

Verify that the vector–valued function defined in (2) solves the two–dimensional system

$$\begin{cases} \frac{dx}{dt} = \lambda_1 x; \\ \frac{dy}{dt} = \lambda_2 y. \end{cases}$$

4. Let

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}, \quad \text{for } t \in \mathbb{R}. \quad (3)$$

Verify that the vector-valued function defined in (3) solves the two-dimensional system

$$\begin{cases} \frac{dx}{dt} = -y; \\ \frac{dy}{dt} = x. \end{cases}$$

5. Let

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

be a solution to the linear system

$$\begin{cases} \frac{dx}{dt} = y; \\ \frac{dy}{dt} = -\omega^2 x, \end{cases}$$

where ω is a positive real number.

Verify that x solves the second order differential equations

$$\frac{d^2x}{dt^2} = -\omega^2 x.$$

Show that y solves the same second order equation.