

## Assignment #9

Due on Wednesday, March 4, 2015

Read Section 4.2.1 on *Fundamental Matrices* in the class lecture notes at <http://pages.pomona.edu/~ajr04747/>

Read Chapter 3, on *Linear Systems*, in Blanchard, Devaney and Hall.

Do the following problems

1. Construct a fundamental matrix for the system  $\begin{cases} \dot{x} = 2y; \\ \dot{y} = x + y. \end{cases}$
2. Construct a fundamental matrix for the system  $\begin{cases} \dot{x} = x + 3y; \\ \dot{y} = 2x + 6y. \end{cases}$
3. Construct a fundamental matrix for the system  $\begin{cases} \dot{x} = 2x + y; \\ \dot{y} = -x + 4y. \end{cases}$
4. Let  $E_A$  denote the fundamental matrix of the two-dimensional linear system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix},$$

where  $A$  is a  $2 \times 2$  matrix with real entries.

Show that  $E_A$  is invertible and

$$[E_A(t)]^{-1} = E_A(-t), \quad \text{for all } t \in \mathbb{R}.$$

5. Let  $A$  and  $E_A$  be as in Problem 4. Show that

$$E_A(t + \tau) = E_A(t)E_A(\tau), \quad \text{for all } t, \tau \in \mathbb{R}.$$