Exam 2 (Part II)

Due on Friday, April 3, 2015

Name: ____________________

This is the out–of–class portion of Exam 2. There are three questions in this portion of the exam. This is a closed–book and closed–notes exam. Show all significant work and give reasons for all your answers.

Students are expected to work individually on these problems. You may not consult with anyone.

Please, write your name on this page and staple it to your solutions. Turn in your solutions at the start of class on **Friday, April 3, 2015.**

I have read and agree to these instructions. Signature: ____________________

1. For the linear, two–dimensional system $\begin{cases} \dfrac{dx}{dt} = 6x + 4y; \\ \\ \dfrac{dy}{dt} = -10x - 6y, \end{cases}$

   (a) compute the fundamental matrix, and give the general solution;

   (b) determine the nature of the stability of the origin, and sketch the phase portrait of system.

2. The following system of first order differential equations can be interpreted as describing the interaction of two species with population densities x and y:

$$\begin{cases} \dfrac{dx}{dt} = x(1 - 2x) - 0.5xy; \\ \\ \dfrac{dy}{dt} = 0.5xy - 0.5y. \end{cases}$$

   (a) What do these equations predict about the population density of each species if the other were not present? What effect do the species have on each other? Describe the kind of interaction that this system models.

   (b) Sketch the nullclines, determine the equilibrium points in the first quadrant, apply the principle of linearized stability (when applicable) to determine the nature of the stability of all the equilibrium points found in part (b), and sketch some possible trajectories.

   (c) Describe the different possible long–run behaviors of x and y as $t \to \infty$, and interpret the result in terms of the populations of the two species.

3. For the second order, linear, homogeneous differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 0,$$

   (a) give the general solution;

   (b) compute the solution that satisfies the initial condition: $x(0) = 1$, $x'(0) = 0$;

   (c) sketch the graph of x as a function of t for the solution found in part (b) in the tx–plane. What happens to $x(t)$ as $t \to \infty$?