Assignment #4

Due on Friday, February 6, 2015

Read Section 3.2, on *Differentiable Paths,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 17.2, on *Motion, Velocity and Acceleration,* in Calculus: Multivariable, by McCallum, Hughes–Hallett, Gleason, et al.

Background and Definitions.

- **Velocity and Acceleration**. A path $\overrightarrow{r}\colon J \to \mathbb{R}^2$ may be used to model the motion of particle in the plane. In this case, $\overrightarrow{r}(t) = (x(t), y(t))$ locates the particle at time t. If x and y are twice differentiable, then

$$\overrightarrow{r}'(t) = (x'(t), y'(t)), \quad \text{for } t \in J,$$

is called the **velocity** of the particle and is usually denoted by $\overrightarrow{v}(t)$. The second derivative,

$$\overrightarrow{r}''(t) = (x''(t), y''(t)), \quad \text{for } t \in J,$$

is called the **acceleration** of the particle and is usually denoted by $\overrightarrow{a}(t)$.

- **Tangent Line to a Curve**. Suppose a curve C is parametrized by a differentiable path $\overrightarrow{r}$. Let $(x_o, y_o) \in C$ be such that $(x_o, y_o) = \overrightarrow{r}(t_o)$, for some $t_o \in J$. Then, the derivative vector

$$\overrightarrow{r}'(t_o) = (x'(t_o), y'(t_o))$$

is tangent to the curve C at (x_o, y_o). The line parametrized by

$$(x_o, y_o) + (t - t_o)\overrightarrow{r}'(t_o), \quad \text{for } t \in \mathbb{R},$$

is the tangent line to C at the point (x_o, y_o).

- **Linear Approximation**. Let $\overrightarrow{r}\colon J \to \mathbb{R}^2$ denote a differentiable path defined on some open interval t_o. Let $t_o \in J$. The expression

$$\overrightarrow{\ell}(t) = (x_o, y_o) + (t - t_o)\overrightarrow{r}'(t_o), \quad \text{for } t \in \mathbb{R},$$

is called the **linear approximation** to the path $\overrightarrow{r}$ at t_o.

Do the following problems

1. A particle moves in the xy–plane along a path determined by the parametric equations
$$\left\{\begin{array}{rcl} x & = & t; \\ y & = & t^3 - t, \end{array}\right. \qquad \text{for } t \in \mathbb{R}.$$
Compute the velocity and acceleration of the particle.

2. A particle moves in the xy–plane along a path determined by the parametric equations
$$\left\{\begin{array}{rcl} x & = & 3\cos t; \\ y & = & 4\sin t, \end{array}\right. \qquad \text{for } t \in \mathbb{R}.$$
Compute the velocity and acceleration of the particle.

3. A particle moves in the xy–plane along a path determined by the parametric equations
$$\left\{\begin{array}{rcl} x & = & 3\cos(t^2); \\ y & = & 4\sin(t^2), \end{array}\right. \qquad \text{for } t \in \mathbb{R}.$$
Compute the velocity and acceleration of the particle.

4. A particle moves in the xy–plane along a path determined by the parametric equations
$$\left\{\begin{array}{rcl} x & = & t\cos t; \\ y & = & t\sin t, \end{array}\right. \qquad \text{for } 0 \leqslant t \leqslant 4\pi.$$
   (a) Sketch the curve parametrized by this path.
   (b) Compute the velocity of the particle at any time $t \in (0, 4\pi)$.

5. A particle moves in the xy–plane along a path determined by the parametric equations
$$\left\{\begin{array}{rcl} x & = & a(t - \sin t); \\ y & = & a(1 - \cos t), \end{array}\right. \qquad \text{for } 0 \leqslant t \in \mathbb{R},$$
where a is a positive number.

   Give the tangent line approximations to the path when t is $\pi/4$ and when $t = \pi$.