Assignment #7

Due on Monday, February 16, 2015

Read Chapter 4, on *Vector Fields,* in the class lecture notes at
`http://pages.pomona.edu/~ajr04747/`

Read Section 17.4, on *The Flow of a Vector Field,* in Calculus: Multivariable, by McCallum, Hughes–Hallett, Gleason, et al.

Background and Definitions.
Flow of a Vector Field. Given a vector field

$$\overrightarrow{F}(x,y) = f(x,y)\widehat{i} + g(x,y)\widehat{j},$$

where f and g are real valued functions defined on a subset of the plane, the flow of $\overrightarrow{F}$ is the set of parametrized curves $(x(t), y(t))$ resulting from solving the pair of equations

$$\begin{cases} \dfrac{dx}{dt} &= f(x,y); \\ \\ \dfrac{dy}{dt} &= g(x,y). \end{cases}$$

Do the following problems

1. Sketch the flow of the vector field

$$\overrightarrow{F}(x,y) = x\widehat{i} - y\widehat{j}.$$

2. Verify that the parametric equations

$$\begin{array}{rcl} x(t) &=& A\cos(t+\phi); \\ y(t) &=& A\sin(t+\phi), \end{array} \qquad \text{for } t \in \mathbb{R},$$

where A and ϕ are constants, are the flow of the vector field

$$\overrightarrow{F}(x,y) = -y\widehat{i} + x\widehat{j}.$$

Sketch the flow of the field.

3. Compute the flow of the field

$$\overrightarrow{F}(x, y) = a\widehat{i} + b\widehat{j},$$

where a and b are constants, and sketch it.

4. Compute the flow of the field

$$\overrightarrow{F}(x, y) = x\widehat{i},$$

and sketch it.

5. Verify that the parametric equations

$$\begin{array}{rcl} x(t) & = & a(e^t + e^{-t}); \\ y(t) & = & b(e^t - e^{-t}), \end{array} \qquad \text{for } t \in \mathbb{R},$$

where a and b are constants, are the flow of the vector field

$$\overrightarrow{F}(x, y) = y\widehat{i} + x\widehat{j}.$$

Sketch the flow of the field.