

Review Problems for Exam 2

1. Put $f(x, y) = 4 - \sqrt{x^2 + y^2}$.
 - (a) Give the domain of f .
 - (b) Sketch a contour plot for the graph of f .
 - (c) Sketch the graph of f .
2. Let $f(x, y) = \sqrt{x^2 + 2y^2}$ for all $(x, y) \in \mathbb{R}^2$. Sketch a contour plot for the function f .
3. Let $f(x, y) = x^2 - y^2$ for all $(x, y) \in \mathbb{R}^2$. Sketch a contour plot for the function f .
4. Give the formula for a linear function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ whose graph contains the points $(1, 4, 7)$, $(4, 7, 0)$ and $(0, 4, 7)$. Sketch the graph of f .
5. Give the equation of plane parallel to the plane $2x + 4y - 3z = 1$ and which goes through the point $(1, 0, -1)$.
6. Compute the first partial derivatives of $f: \mathbb{R}^2 \rightarrow \mathbb{R}$ given by $f(x, y) = (4x - x^7 - y)^4$ for all $(x, y) \in \mathbb{R}^2$.
7. Give the equation of the tangent plane to the graph of $z = ye^{x/y}$ at the point $(1, 1, e)$.
8. Compute the differential of f , where $f(x, y) = \sqrt{x^2 + y^3}$, for all $(x, y) \in \mathbb{R}^2$, at the point $(1, 2)$, and use it to estimate $f(1.04, 1.98)$.
9. Assume that the temperature, $T(x, y)$, at a point (x, y) in the plane is given by
$$T(x, y) = \frac{100}{1 + x^2 + y^2}, \quad \text{for all } (x, y) \in \mathbb{R}^2.$$
 - (a) Sketch the contour plot for T .
 - (b) Locate the hottest point in the plane. What is the temperature at that point?
 - (c) Give the direction of greatest increase in temperature at the point $(3, 2)$. What is the rate of change of temperature in that direction?
 - (d) A bug moves in the plane along a path given by $\vec{r}(t) = t\hat{i} + t^2\hat{j}$ for $t \in \mathbb{R}$. How fast is the temperature changing when $t = 1$.
10. Let $f(x, y) = x^2 + y^2$ for all $(x, y) \in \mathbb{R}^2$. Sketch the flow of the vector field $\nabla F(x, y)$.