Assignment #9

Due on Wednesday, March 1, 2017

Read Section 4.2.1 on *Fundamental Matrices* in the class lecture notes at `http://pages.pomona.edu/~ajr04747/`.

Read Section 3.1 on *Properties of Linear Systems and the Linearity Principle* in Blanchard, Devaney and Hall.

Do the following problems

1. Construct a fundamental matrix for the system $\begin{cases} \dot{x} &= 2y; \\ \dot{y} &= x + y. \end{cases}$

2. Construct a fundamental matrix for the system $\begin{cases} \dot{x} &= x + 3y; \\ \dot{y} &= 2x + 6y. \end{cases}$

3. Construct a fundamental matrix for the system $\begin{cases} \dot{x} &= 2x + y; \\ \dot{y} &= -x + 4y. \end{cases}$

4. Let $E_{_A}$ denote the fundamental matrix of the two–dimensional linear system

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix},$$

   where A is a 2×2 matrix with real entries.

   Show that $E_{_A}$ is invertible and

$$[E_{_A}(t)]^{-1} = E_{_A}(-t), \quad \text{for all } t \in \mathbb{R}.$$

5. Let A and $E_{_A}$ be as in Problem 4. Show that

$$E_{_A}(t + \tau) = E_{_A}(t)E_{_A}(\tau), \quad \text{for all } t, \tau \in \mathbb{R}.$$